Numerical Integration
Trapezoidal Rule

Objectives

- Numerically integrate a function using Trapezoidal Rule
- Trapezoidal rule falls under the closed form of the Newton-Cotes integration formulas.
- Newton-Cotes formulas are the most common integration schemes.
- Here we replace a complicated function or tabulated data using an approximating function that is easy to integrate.

Newton – Cotes Formula

- $I = \int_a^b f(x)\,dx \sim = \int_a^b f_n(x)\,dx$
- $f_n(x)$ – polynomial of the form
- $f_n(x) = a_0 + a_1 * x + \ldots\ldots + a_{n-1} * x^{n-1} + a_n * x^n$
- n – order of the polynomial
- n = 1 in Fig 1 (1st order polynomial - Straight line)
- n = 2 in Fig 2 (2nd order polynomial - Parabola)

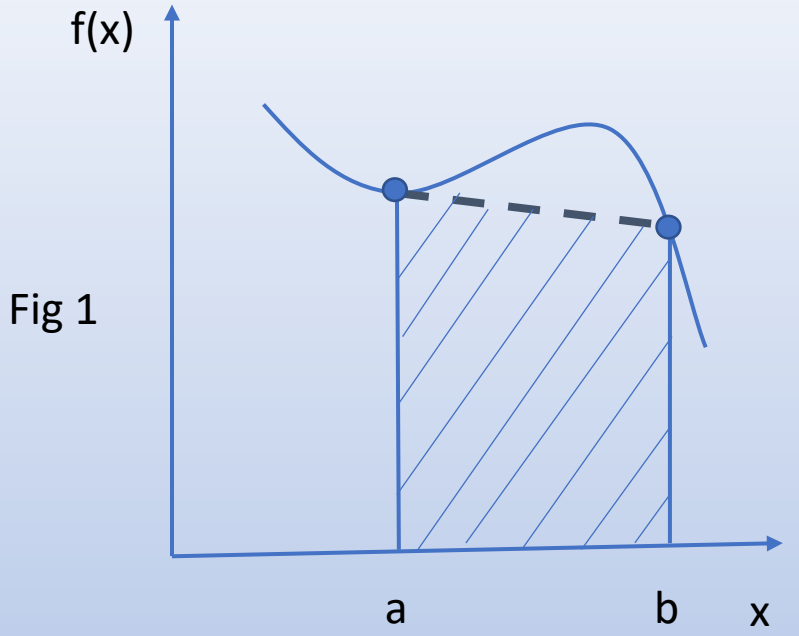

Fig 1

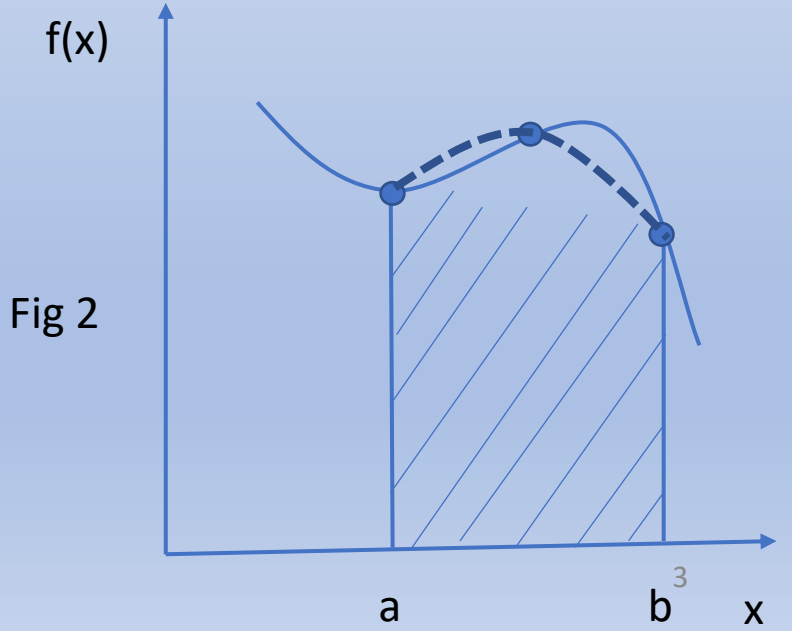

Fig 2

Trapezoidal Rule

- n = 1 in Fig 1 (1st order polynomial - Straight line)
- The trapezoidal rule for a single interval is obtained by fitting a first-degree polynomial to two discrete points
- $I = \int_a^b f(x)\,dx = \int_a^b f_1(x)\,dx$
- $f_1(x) = f(a) + \frac{(f(b)-f(a))}{(b-a)} * (x-a)$ (using laws of similar triangles from Fig 1)
- $I = \int_a^b f_1(x)\,dx = \int_a^b [\, f(a) + \frac{(f(b)-f(a))}{(b-a)} * (x-a)\,]\,dx$
- $I = (b-a) * \frac{(f(a)+f(b))}{2}$

Trapezoidal Rule

- Individual error, $Et = -\frac{(b-a)^3}{(12n^2)} * (f_m'')$; $f_m'' = \frac{\sum_{i=1}^{n} f''(\xi i)}{n}$; ξi is a point in segment i
- Global error – $O(h^2)$; h = interval size;
- n – no of segments
- If n is doubled, the error term becomes one quarter of the original error value
- So to reduce the error term, the no. of segments, n need to be increased,

Trapezoidal Rule

- Example – Integrate the function
- f(x) = 0.2 + 25*x - 200 * x^2 + 675*x^3 - 900*x^4 + 400*x^5;
- From a = 0 to b = 0.8
- The value obtained analytically is 1.640533
- Evaluate the integral by varying the no. of segments, n

Summary

In this video,

- We presented Trapezoidal rule to numerically integrate a function f(x).
- Trapezoidal rule is not a very efficient method.
- The error is $O(h^2)$.
- In the next video we can look at other methods such as Simpson's 1/3 and 3/8 rules.